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Coupling of wall conduction with natural convection in a rectangular enclosure

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Abstract—Conjugate heat transfer has been studied in an enclosure which consists of a conducting vertical wall of finite thickness with a uniform heat input, an insulated vertical wall and two horizontal walls at a heat sink temperature. The relative heat removal contribution by conduction in the solid wall to that by natural convection in the fluid enclosure, as well as the temperature and temperature gradient on the fluid–solid interface have been obtained for Rayleigh numbers, Ra, between 10^2 and 10^7, solid–fluid conductivity ratio, R_k, between 0.5 and 50, dimensionless solid wall thickness, W, between 0.05 and 0.25 and enclosure aspect ratio, A, between 0.1 and 10. The results indicate that for low Ra, high R_k and W at small and large A, the heat transfer process is dominated by the heat conduction in the solid wall. For high Ra, low R_k and W at moderate A, strong interaction between conduction in the solid wall and convection in the fluid influences the heat transfer.

INTRODUCTION

CONJUGATE heat transfer is involved in many practical problems where convective heat transfer occurs on the surfaces of a conducting solid wall of finite thickness. Since the early assertion by Luikov *et al.* [1] on the necessity of taking into account wall conduction in such problems, numerous studies on the coupling of solid wall conduction with fluid convection have been reported in the literature. In relation to the present work, Koutsoheras and Charters [2] investigated, numerically, the natural convection in a rectangular enclosure heated from the bottom, with the left wall being a conducting wall of finite thickness. Meyer *et al.* [3] studied a similar problem but included some experimental results to confirm their numerical computation. Kim and Viskanta [4] in their computational study considered not only the effect of wall conduction, but also wall radiation in a rectangular enclosure with all four walls being conducting solid walls of finite thickness. They also presented further detailed experimental results on the influence of wall conduction for similar geometry [5].

The present study is motivated by electronic cooling applications. Heat producing electronic components are often mounted on a printed circuit board (PCB) above a conducting plate. The heat produced is then transferred, both by conduction through the plate to its two ends and by natural convection in the surrounding fluid to the heat sinks. As a result, the heat removing rate from the electronic components will depend on the coupling of the wall conduction and the fluid convection. It is of interest to understand when this coupling is important, since it will directly influence the temperature distribution among the components and thus the design of heat removing mechanisms in practical applications.

PHYSICAL MODEL AND NUMERICAL METHOD

The physical model considered is shown in Fig. 1. A uniform heat flux is exerted on the left wall of a rectangular enclosure which is of conductivity k_p and thickness w. The right wall of the enclosure is assumed to be insulated. The two horizontal walls are at heat sink temperature T_c. Other relevant geometrical dimensions are given in the figure.

The numerical analysis of the physical situation depicted in Fig. 1 was carried out by using a two-dimensional model, developed earlier by the present authors, for studying natural convection in enclosures which may be partially divided and may contain partitions of various geometries [6]. The flow is assumed to be steady, laminar and the Boussinesq approximation is used to account for the density variation. The mathematical model is briefly described below.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\mu\nabla^2 u \tag{2}$$

$$\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\mu\nabla^2 v+\rho g\beta(T-T_c) \tag{3}$$

$$\rho C_p u\frac{\partial T}{\partial x}+\rho C_p v\frac{\partial T}{\partial y}=k\nabla^2 T. \tag{4}$$

Boundary conditions are (see Fig. 1)

$$u=v=0 \qquad \text{on the walls of the enclosure}$$

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NOMENCLATURE

A	aspect ratio, H/L
C_p	heat capacity [J kg^{-1} K^{-1}]
g	acceleration due to gravity [m s^{-2}]
H	cavity height [m]
k	thermal conductivity [W m^{-2} K^{-1}]
L	cavity width including solid wall [m]
L_f	fluid cavity width [m]
p	pressure, $p_{atm}+qgy$ [Pa]
P	dimensionless pressure, $pL^2/(q\alpha^2)$
Pr	Prandtl number, m/α
Q	dimensionless heat flux defined by equations (9) and (10)
q	heat flux [W m^{-2}]
q_y	dimensionless heat flux at solid–fluid interface
R_k	dimensionless conductivity ratio, k_p/k_f
Ra	Rayleigh number, $g\beta\Delta T_{max}L^3/(m\alpha)$
T	temperature [K]
u, v	fluid velocity in x and y directions [m s^{-1}]
U	dimensionless fluid velocity in x direction, uL/α
V	dimensionless fluid velocity in y direction, vL/α
x, y	Cartesian coordinates
X	dimensionless distance on x axis, x/L
Y	dimensionless distance on y axis, y/L
W	dimensionless solid wall thickness, w/L
w	solid wall thickness [m].

Greek symbols

α	thermal diffusivity, $k/(qc_p)$ [m^2 s^{-1}]
β	thermal expansion of fluid [K^{-1}]
ΔT_{max}	temperature difference, qL/k_f
η	relative heat removal factor, Q_{ts}/Q_t
Θ	dimensionless temperature, $(T-T_c)/\Delta T_{max}$
λ	dimensionless dynamic viscosity, μ_p/μ_f
μ	dynamic viscosity [kg m^{-1} s^{-1}]
ν	kinematic viscosity [m^2 s^{-1}]
ρ	fluid density [kg m^{-3}].

Subscripts

c	cold wall
f	fluid
h	hot wall
p	solid
t	top horizontal bounding wall with width L
ts	solid portion of the top horizontal wall with width w
1	left wall
2	right wall.

q = constant on $x = 0$, $y = 0$–H

$\dfrac{\partial T}{\partial x} = 0$ on $x = L$, $y = 0$–H

$T = T_c$ on $x = 0$–L, $y = 0$ and H.

The non-dimensional equations are obtained by using non-dimensional parameters defined in the nomenclature as

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{5}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\lambda\,Pr\nabla^2U \tag{6}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\lambda\,Pr\nabla^2V+Pr\,Ra\,\Theta \tag{7}$$

$$U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}=R_k\nabla^2\Theta \tag{8}$$

where λ and R_k are equal to 1 in the fluid region and $\lambda = 10^{15}$, $R_k = k_p/k_f$ in solid region.

Based on the above model, the dimensionless heat flux leaving from the top horizontal bounding wall is defined as

$$Q_t=\frac{1}{A}\int_0^1\left.\frac{\partial\Theta}{\partial Y}\right|_{Y=A}\mathrm{d}X \tag{9}$$

while the heat flux leaving from the solid portion of the top horizontal bounding wall is

$$Q_{ts}=\frac{1}{A}\int_0^W\left.\frac{\partial\Theta}{\partial Y}\right|_{Y=A}\mathrm{d}X. \tag{10}$$

The relative heat removal contribution by conduction in the solid wall from the top horizontal wall is thus defined as

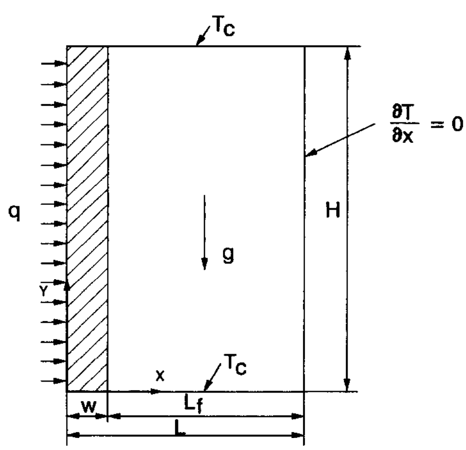

Fig. 1. Enclosure with left conducting solid wall of thickness W.

$$\eta_t = \frac{Q_{ts}}{Q_t}. \tag{11}$$

The dimensionless heat flux on the solid interface is defined by

$$q_y(X_p) \equiv \frac{-k_f \left.\dfrac{\partial T}{\partial X}\right|_{X_p^+}}{q} = \frac{-k_p \left.\dfrac{\partial T}{\partial X}\right|_{X_p^-}}{q} \tag{12}$$

Obviously, for the present study, Q_t, η_t, and $q_y(X_p)$ will directly reflect the intensity of the interaction between conduction in the solid wall and convection in the fluid. A higher Q_t, lower η_t and higher $q_y(X_p)$ will imply a more intensive interaction in the system.

The numerical solution method used is the SIMPLER method [7]. Details of the program have been presented elsewhere [6] and will not be repeated here. Three different mesh sizes of 25×25, 40×40 and 80×80 have been used in the study. The finest size 80×80 was used for cases with a large aspect ratio A when the effective heat transfer surface, i.e. the two horizontal walls are relatively small. In all the computations, the energy balance within the system was kept at 0.1% combined with mass conservation within 10^{-3} (0.5×10^{-2} was used for finest mesh size 80×80 since the grid number is greatly increased in this situation).

Typical number of iterations on IBM 3090 with 17 mips was about 50. For the cases with aspect ratio $A > 2$, the accuracy criterion was achieved using iterations above 100.

RESULTS AND DISCUSSION

To understand the interaction between conduction in the solid wall and convection in the fluid, numerical computation has been generated for different combinations of Ra, R_k, W and A in respect to their influences on Q_t, η_t, $q_y(X_p)$ and the interface temperature distribution. Pr was taken as 0.71 for all cases studied.

Figure 2 shows the influence of Ra on Q_t and η_t for three R_k values of 1, 10 and 50. With increasing Ra, Q_t generally increases since higher Ra means more intensive convection in the fluid enclosure. This corresponds to the decrease of η_t. However, this tendency of Q_t increasing with Ra slows down for higher R_k. As indicated in the figure, when $R_k = 50$, almost all the heat is removed by conduction in the solid portion of the horizontal wall and η_t is about 90%. This means little heat is transferred into the fluid for stimulating natural convection. Thus Q_t is kept around 0.5 for $R_k = 50$. This should be so, since in the conduction limit, the heat removed from the top and bottom horizontal walls will be equal while the total dimensionless heat input is 1.0.

The above overall results can be further explored by the temperature and heat flux distributions on the solid–fluid interface shown in Figs. 3 and 4. For

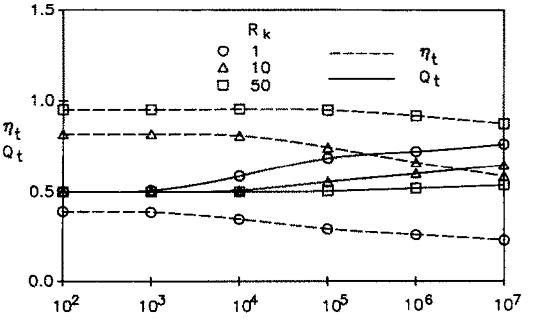

FIG. 2. Influence of Ra on η_t and Q_t for different R_k with $A = 1$, $W = 0.1$.

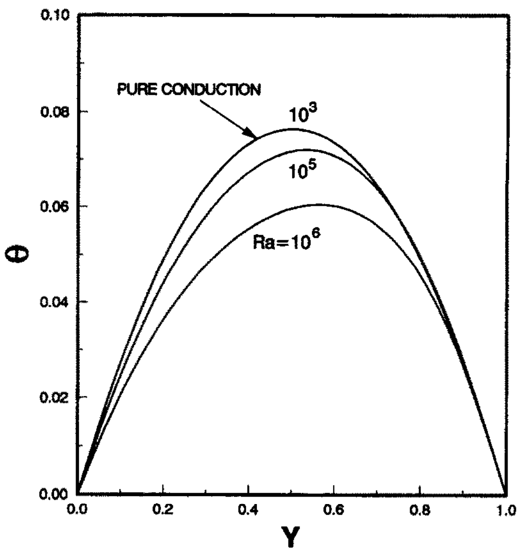

FIG. 3. Temperature variation along Y on solid–fluid interface for different Ra with $R_k = 10$, $A = 1$ and $W = 0.1$.

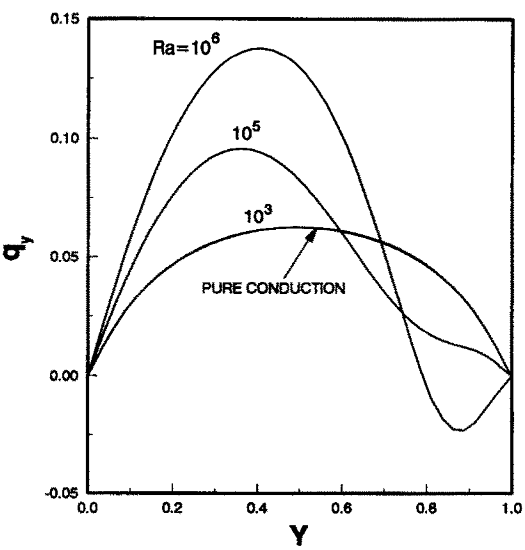

FIG. 4. Local heat flux along Y on solid–fluid interface for different Ra with $R_k = 10$, $A = 1$ and $W = 0.1$.

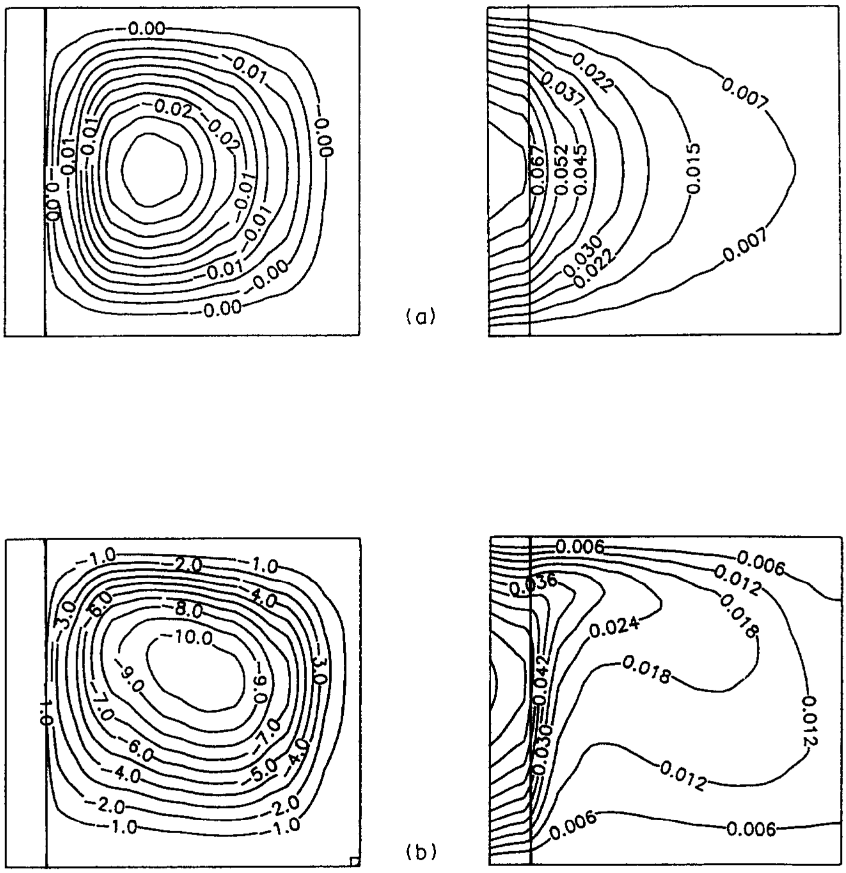

Fig. 5. Isotherms and streamlines for (a) $Ra = 10^3$ and (b) $Ra = 10^6$ with $R_k = 10$, $A = 1$ and $W = 0.1$. Streamlines are on the left and isotherms on the right.

$Ra = 10^3$, which is almost identical to the case for pure conduction, a sine-like heat flux is present on the interface as there is no convection in the fluid. Correspondingly, a sine-like temperature distribution appears on the interface. However, when Ra increases to 10^6, strong interaction between wall conduction and fluid convection is observed. The higher Ra means more heat input q, and as a result more heat is added to the fluid to intensify the fluid convection. The intensified fluid convection, in turn, enables more heat to be received by the fluid through convective heat transfer on the interface. This results in an increase of Q_t and a decrease of η_t as shown in Fig. 2. With the intensifying of convection in the fluid, more heat is brought to the upper portion of the fluid enclosure. In the mean time, the solid wall temperature decreases due to heat loss by the intensified convection. Therefore, in the upper portion of the enclosure, the fluid is at a higher temperature than the solid wall and heat is transferred from the fluid to solid wall. This phenomenon can be examined in Fig. 5 where the flow and temperature fields are produced for $Ra = 10^3$ and 10^6. It can be seen that in Fig. 5(b), the fluid temperature is higher near the left upper part than that of the solid wall. The same is not observed in Fig. 5(a) for $Ra = 10^3$. This phenomenon for $Ra = 10^6$ results also in the negative $q_y(X_p)$ shown in Fig. 4. Further, a non-sine like temperature distribution on the interface space is also present.

The variation of Q_t and η_t as a function of R_k is shown in Fig. 6. With increasing R_k, $Q_t \approx 0.5$ and $\eta_t > 85\%$. This suggests that the heat is mainly removed by the solid wall conduction. Only less than 15% heat removal is attributed to the fluid convection.

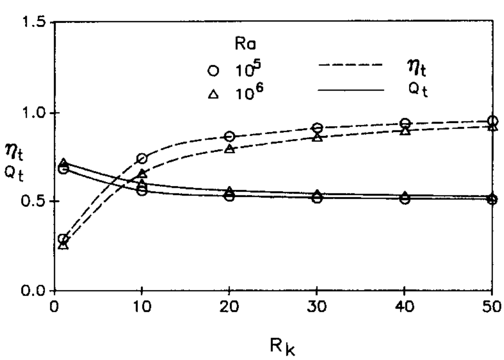

Fig. 6. Influence of R_k on η_t and Q_t for $Ra = 10^5$ and 10^6 with $A = 1$, $W = 0.1$.

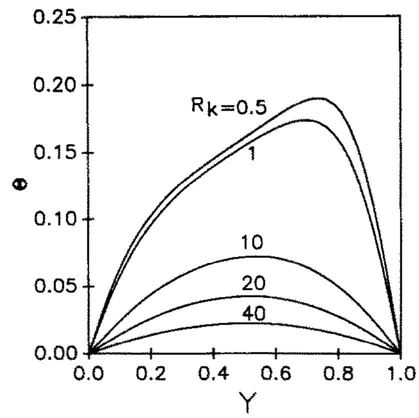

FIG. 7. Temperature variation along Y on solid–fluid interface for different R_k with $Ra = 10^5$, $A = 1$ and $W = 0.1$.

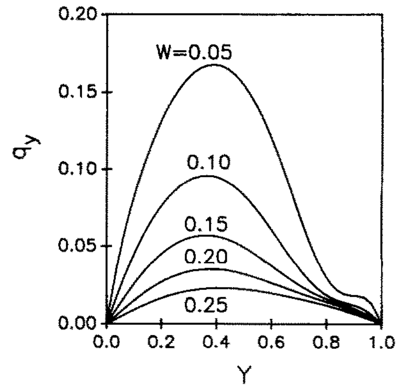

FIG. 9. Local heat flux along Y on solid–fluid interface for different W with $Ra = 10^5$, $R_k = 10$ and $A = 1$.

This means that for practical applications, if the conductivity of the PCB plate is relatively high, natural convection will not be a significant concern.

The above phenomena can be further examined by studying the effect of R_k, which is presented in Figs. 7 and 8. When the solid wall possesses the same conductivity as that of the fluid, i.e. $R_k = 1$, the solid wall presents no advantage over the fluid on heat transfer. As a result, due to the small thickness of the wall, only little heat is transferred longitudinally in the solid wall to its two ends while most heat is transferred to the fluid. Thus, the convection in the fluid is intensified and a strong non-sine like higher temperature distribution $\Theta(X_p)$ and a higher heat flux $q_y(X_p)$ become present on the solid–fluid interface as shown in Figs. 7 and 8. As a result, a higher Q_t and lower η_t are observed in Fig. 6.

When R_k increases, the situation becomes different. As indicated in Fig. 7, when $R_k > 10$, due to higher conductivity of the solid wall, the heat is effectively transferred longitudinally in the solid wall to its two ends and less heat is received by the fluid. Therefore, the significance of natural convection is reduced. The influence of the fluid natural convection on the solid wall conduction process is also diminished. A sine-like temperature distribution is followed on the interface, as shown in Fig. 7. Also, a much lower interface heat flux $q(X_p)$ is observed in Fig. 8. When $R_k = 40$, $q_y(X_p)$ on the interface is so low that only little heat is transferred to the fluid for stimulating natural convection. As seen in Fig. 6, for the same conditions as Fig. 8, the amount of heat removed is about 95% by solid wall conduction and the rest by convection.

The solid wall thickness W also influences the coupling between conduction in solid wall and convection in fluid in a similar way to R_k. In fact, a larger W has the same effect as a larger R_k on increasing longitudinal conduction in the solid wall and thus less convection in fluid. As shown in Figs. 9 and 10, with increasing W, a lower $q_y(X_p)$ is observed on the solid–fluid interface. Also, the temperature on the interface approaches a sine-like distribution which implies less of an influence of fluid convection on the solid wall heat conduction.

The influence of the fluid enclosure aspect ratio A is shown in Figs. 11 and 12. When A is small, such as $A \leqslant 1$, Q_t is close to its conduction limit value of 0.5 and η_t higher than 90% regardless of R_k value. This phenomenon can be explained as follows: as shown in Fig. 12, for a small aspect ratio such as $A = 1.0$,

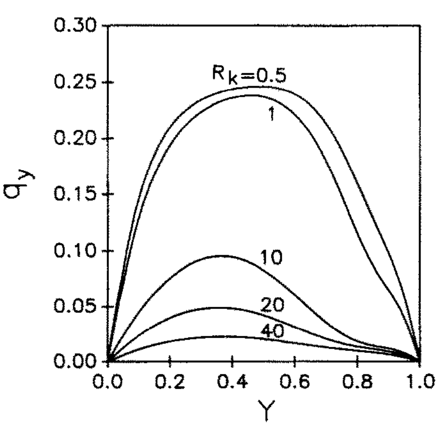

FIG. 8. Local heat flux along Y on solid–fluid interface for different R_k with $Ra = 10^5$, $A = 1$ and $W = 0.1$.

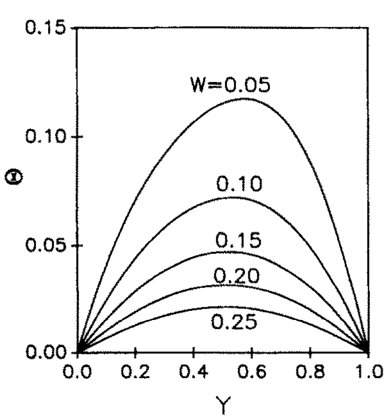

FIG. 10. Temperature variation along Y on solid–fluid interface for different W with $Ra = 10^5$, $R_k = 10$ and $A = 1$.

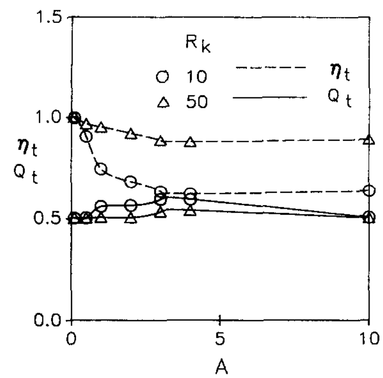

FIG. 11. Influence of A on η_t and Q_t for R_k = 10, 50 with $Ra = 10^5$, $W = 0.1$.

heat can be easily conducted longitudinally in the solid wall to its two ends due to the relative short conduction length. Thus only little heat is transferred to the fluid for convection and a lower Q_t and higher η_t result. Similarly, lower Q_t and higher η_t are also observed for a large aspect ratio of $A = 10$ in Fig. 11. However, the physical situation is different from the case of small A. For $A = 10$, as indicated in Fig. 12, much more heat is transferred to the fluid than in the case of $A = 1$. The reason is that a larger A makes heat difficult to be transferred by longitudinal conduction in the solid wall to its two ends. However, even if the fluid receives more heat, the narrow fluid enclosure strongly restricts the development of natural convection. Thus, the heat transfer mechanism in the fluid is dominated by conduction. Since the conductivity of the solid wall is much higher than that of the fluid ($R_k = 10$), more heat is transferred to heat sinks by solid wall conduction. This results in the lower Q_t and higher η_t in Fig. 11.

It is therefore easier to understand that a maximum Q_t and a minimum η_t exist for a moderate aspect ratio of about $A = 4$ in Fig. 11. For this case, even though the solid wall height is much smaller than that for $A = 10$, and that the heat is transferred easier by conduction longitudinally in the solid wall to its two ends, almost the same amount of heat is transferred to the fluid. This is due to the strong interaction between conduction in the solid wall and convection in fluid. As shown in Fig. 12, a negative $q_y(X_p)$ is again present on the upper portion of the interface which implies a net heat transfer from fluid to solid wall. The same negative $q_y(X_p)$ has been encountered on the interface in Fig. 4 where a very large Ra was involved which intensified the natural convection and the interaction between conduction and convection. The same is true for the case of $A = 4$ considered here. The moderate aspect ratio provides the most favorable condition to intensify the natural convection and thus exerts more influence on the conduction heat transfer within the solid wall.

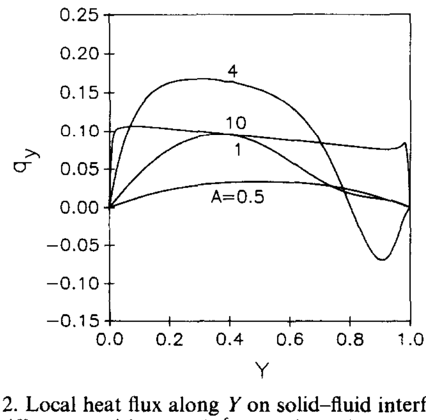

FIG. 12. Local heat flux along Y on solid–fluid interface for different A with $Ra = 10^5$, $R_k = 10$ and $W = 0.1$.

CONCLUSION

For an enclosure with a heated conducting side wall, this study indicates that heat transfer is strongly influenced by the coupling effect between solid wall conduction and fluid convection. For a large conductivity ratio R_k and solid wall width W, heat transfer is dominated by conduction in the solid wall. While for lower R_k and W, increased fluid convection enables more heat to be transferred to fluid and the fluid convection is further intensified. This coupling phenomenon is observed more clearly for higher Ra cases when more heat is added into fluid to increase convection. The increased convection augments the convective heat transfer on the solid–fluid interface which results in more heat being received by the fluid to again intensify the fluid convection. This strong coupling effect, between solid wall conduction and fluid convection, even produces a pure heat flux from fluid to solid wall on the upper portion of the solid–fluid interface. It is noted that the temperature distribution on the interface is greatly influenced by the coupling effect. The coupling effect is also found to be significant for enclosures of moderate aspect ratio A.

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COUPLAGE DE LA CONDUCTION DANS LA PAROI AVEC LA CONVECTION NATURELLE DANS UNE CAVITE RECTANGULAIRE

Résumé—On étudie le transfert thermique conjugué dans une cavité comprenant une paroi verticale conductrice d'épaisseur finie avec une entrée uniforme de chaleur, une paroi verticale isolée et deux parois horizontales à une température de puits de chaleur. La contribution d'enlèvement de chaleur par conduction dans la paroi solide comparée à celle par convection naturelle du fluide, aussi bien que la température et le gradient de température à l'interface fluide–solide ont été déterminés pour un nombre de Rayleigh Ra entre 10^2 et 10^7, un rapport de conductivité solide–fluide R_k entre 0,5 et 50, une épaisseur adimensionnelle de paroi W entre 0,05 et 0,25 et un rapport de forme de la cavité A entre 0,1 et 10. Les résultats indiquent que pour Ra faible, R_k et W élevés, pour A petit et grand, le mécanisme de transfert thermique est dominé par la conduction thermique dans la paroi solide. Pour Ra élevé, R_k et W faible, à A modéré, une forte interaction entre la conduction dans la paroi solide et la convection dans le fluide influence le transfert de chaleur.

KOPPLUNG ZWISCHEN DER NATÜRLICHEN KONVEKTION IN EINEM RECHTECKIGEN HOHLRAUM UND DER WÄRMELEITUNG IN DESSEN WAND

Zusammenfassung—Es wird der konjugierte Wärmetransport in einem Hohlraum untersucht, der aus einer wärmeleitenden senkrechten Wand endlicher Dicke mit einer gleichförmigen Wärmezufuhr, einer isolierten senkrechten Wand und zwei waagerechten Wänden als Wärmesenke konstanter Temperatur besteht. Der relative Anteil der Wärmeübertragung durch Leitung in der Wand im Vergleich zum konvektiven Anteil wurde zusammen mit der Temperatur und dem Temperaturgradienten an der Fluid-Feststoff-Grenzfläche ermittelt und zwar für folgende Parameter: Rayleigh-Zahl (Ra) zwischen 10^2 and 10^7, Verhältnis R_k der Leitfähigkeiten von Feststoff und Fluid zwischen 0,5 und 50, dimensionslose Wandstärke W zwischen 0,05 und 0,25 sowie Seitenverhältnis A des Hohlraums zwischen 0,1 und 10. Die Ergebnisse zeigen, daß für kleine Werte von Ra, große Werte für R_k und W (bei kleinen und großen A) der Wärmetransportvorgang durch die Wärmeleitung in der festen Wand bestimmt wird. Für große Werte von Ra, kleine Werte von R_k und W (bei mittlerem A) besteht eine starke Wechselwirkung zwischen der Leitung in der festen Wand und der Konvektion des Fluids im Hinblick auf den Wärmeübergang.

ТЕПЛОПЕРЕНОС ПРИ НАЛИЧИИ ТЕПЛОПРОВОДЯЩЕЙ СТЕНКИ И ЕСТЕСТВЕННОЙ КОНВЕКЦИИ В ПОЛОСТИ ПРЯМОУГОЛЬНОГО СЕЧЕНИЯ

Аннотация—Исследуется сопряженный теплоперенос в полости, состоящей из теплопроводящей вертикальной стенки конечной толщины с однородным подводом тепла, изолированной вертикальной и двух горизонтальных стенок, через которые осуществляется сток теплового потока. Определяются относительный вклад теплоотвода за счет теплопроводности твердой стенки по сравнению с теплоотводом за счет естественной конвекции в заполненной жидкостью полости, а также температура и температурный градиент на границе раздела жидкость–твердое тело для чисел Рэлея Ra, изменяющихся от 10^2 до 10^7, отношений теплопроводностей твердого тела и жидкости R_k-от 0,5 до 50, безразмерной толщины твердой стенки W-от 0,05 до 9,25 и отношеъий сторон полости A-от 0,1 до 10. Полученные результаты показывают, что в случае низких значений Ra и высоких R_k и W при малых и больших значениях A процесс теплопереноса определяется теплопроводностью в твердой стенке. В случае высоких значений Ra и низких R_k и при умеренных значениях A на теплоперенос влияет сильное взаимодействие между теплопроводностью в твердой стенке и конвекцией в жидкости.